
MATHCOUNTS®

2012

■ National Competition ■
Sprint Round with ANSWERS
Problems 1–30

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2012 MATHCOUNTS
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1. 21 Hiram is helping his little sister write her integers from 1 to 50. He suggests that they take a break after she has written her 33rd digit. What was the last two-digit number that Hiram's little sister wrote before they took the break?

2. 66 degrees The 660 students at Mandelbrot Middle School voted on their choice for favorite among six mathematicians. The table shows the results of the vote. Finn made a circle graph to represent the data in the table. In degrees, what is the measure of the central angle of the sector that represents votes for Gauss?

Mathematicians	Number of Votes
Euclid	100
Gauss	121
Germain	200
Hypatia	48
Pascal	66
Pythagoras	125

3. 44 What is the sum of all positive integers n such that $\sqrt{3} \leq n \leq \sqrt{99}$?

4. 3 sets Ten items have the following weights, in pounds: 2, 3, 5, 5, 6, 6, 8, 8, 8, 9. The items are then divided into three sets of 20 pounds each. How many sets contain an 8 pound weight?



5. 36 The sum of five different positive integers is 80. What is the largest possible value of the second largest number?

6. \$ 8.00 per hr
or 8 From the time a shop opens at 10 a.m., one customer enters the shop every 15 minutes, until the shop closes at 7 p.m. There is a $\frac{1}{3}$ chance that the salesman will convince the customer to buy a widget. The shop owner makes a profit of \$6 on each widget sold. What is the most the shop owner can pay the salesman (in dollars per hour) to exactly break even? That is, what hourly rate will make the amount paid to the salesman equal the total amount of income from widget sales?

7. $\frac{17}{32}$

The odds against event A occurring are 3:1 and the odds in favor of event B occurring are 3:5. What is the probability that at least one of these independent events occurs? Express your answer as a common fraction.

8. $18,000$ people

The population of Expotown is currently 54,000 people. If the population has increased by 200% in the past 30 years, what was the population 30 years ago?

9. 52

If $a + 2b + 3c = 30$ and $a + 3b + 5c = 8$, what is the value of $a + b + c$?

10. 11 ways

A bag contains ten identical blue marbles and ten identical green marbles. In how many distinguishable ways can five of these marbles be put in a row if there are at least two blue marbles in the row and every blue marble is next to at least one other blue marble?



11. 4 integers

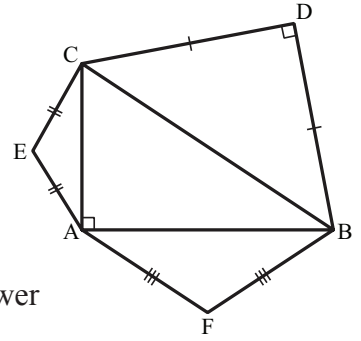
For how many integers, n , in $\{1, 2, \dots, 20\}$ is the tens digit of n^2 odd?

12. $-\frac{4}{11}$

Line L containing the point $(-11, -4)$ has the same y -intercept as $y = -3x - 2$. What is the product of the slope and y -intercept of line L? Express your answer as a common fraction.

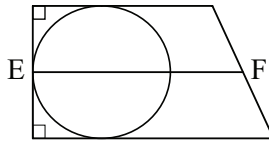
13. $\frac{\sqrt{3}}{3}$

Triangle ABC is a right triangle with AB = 8 units, BC = 10 units and AC = 6 units. Triangles BDC, AFB and CEA are isosceles with vertex angles of 90° , 120° and 120° , respectively. If R denotes the area of $\triangle CEA$, S denotes the area of $\triangle AFB$ and T denotes the area of $\triangle BDC$, what is the value of $\frac{R+S}{T}$? Express your answer as a common fraction in simplest radical form.



14. 54 units²

In the figure, a circle is located inside a trapezoid with two right angles so that a point of tangency of the circle is the midpoint of the side perpendicular to the two bases. The circle also has points of tangency on each base of the trapezoid. The diameter of the circle is $\frac{2}{3}$ the length of segment EF, as shown. If the area of the circle is 9π units², what is the area of the trapezoid?



15. 3

The sum of the reciprocals of three consecutive positive integers is equal to 47 divided by the product of the integers. What is the smallest of the three integers?

16. 243

The first three terms of a geometric sequence are 3, $6 + p$ and $30 - p$, and each term is a positive number. What is the fifth term of this sequence?

17. $\frac{7}{72}$

A standard die whose faces are numbered 1, 2, 3, 4, 5, 6 is rolled three times. What is the probability that the sum of the numbers rolled is 8? Express your answer as a common fraction.

18. $\frac{49}{6}$

If x , y and z satisfy $xy = 1$, $yz = 2$ and $xz = 3$, what is the value of $x^2 + y^2 + z^2$? Express your answer as a common fraction.

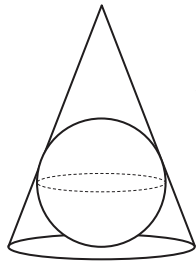
19. $10\sqrt{5}$ feet

A cubical room has edge length 10 feet with A and B denoting two corners that are farthest apart. A caterpillar crawls from A to B along the walls. In feet, what is the length of the shortest path from A to B that the caterpillar may have taken? Express your answer in simplest radical form.

20. 2.5

What is the maximum value of xy if $x + \frac{1}{y} = \frac{7}{2}$ and $y + \frac{1}{x} = \frac{7}{5}$? Express your answer as a decimal to the nearest tenth.

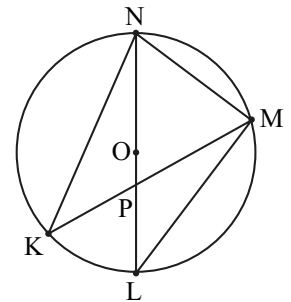
21. 14.4 inches



A sphere of radius 4 inches is inscribed in a cone with a base of radius 6 inches. In inches, what is the height of the cone? Express your answer as a decimal to the nearest tenth.

22. 12 units

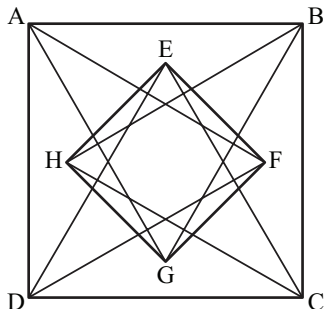
In circle O, shown, $OP = 2$ units, $PL = 8$ units, $PK = 9$ units and $NK = 18$ units. Points K, P and M are collinear, as are points L, P, O and N. What is the length of segment MN?



23. 799

Consider the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 19, 29, ..., where the sum of the digits of a_n , the n th term, is equal to n , and a_{n+1} is the smallest positive integer such that $a_{n+1} > a_n$. What is the value of the 25th term of this sequence?

24. 2 - √3



Four equilateral triangles, $\triangle ABG$, $\triangle BCH$, $\triangle CDE$ and $\triangle DAF$, are constructed inside square $ABCD$, as shown. Points E , F , G and H are the vertices of the triangles that lie within square $ABCD$. What is the ratio of the area of square $EFGH$ to the area of square $ABCD$? Express your answer in simplest radical form.

25. 26 players

In a round-robin chess tournament every player plays one game with every other player. Five participants withdrew after playing two games each. None of these players played a game against each other. A total of 220 games were played in the tournament. Including those who withdrew, how many players participated?

26. 5/8

The base-three repeating decimal $0.\overline{12}$ is equivalent to what base-ten common fraction?

27. 6

An arithmetic series of positive integers has 8 terms and a sum of 2008. What is the smallest possible value of any member of the series?

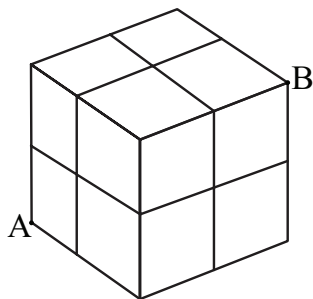
28. 225 numbers

How many whole numbers n , such that $100 \leq n \leq 1000$, have the same number of odd factors as even factors?

29. 4

If x and y satisfy $(x - 3)^2 + (y - 4)^2 = 49$, what is the minimum possible value of $x^2 + y^2$?

30. 54 short paths



Eight identical unit cubes are stacked to form a $2 \times 2 \times 2$ cube, as shown. A “short path” from vertex A to vertex B is defined as one that consists of six one-unit moves either right, up or back along any of the six faces of the 2 -unit cube. How many “short paths” are possible?

